CS5229 HW1

1. You roll two fair dice and sum up the values shown on the top of the two dice. What is the expected value of this sum?

A: roll one dice (1+2+3+4+5+6)/6 = 3.5 , the 2 dice are independent, E(X+Y)= E(X)+E(Y) if X, Y are independent.

Sum = 3.5 +3.5 = 7

1. Customers arrive at a fast food restaurant at a rate of five per minute (assume Poisson arrivals) and wait to receive their order for an average of 5 minutes.

a. (10pt) How many customers are there in the restaurant on the average?

λ= 5, T = 5

N = λ \* T = 5 \* 5 = 25

b. (10pt) If customers eat in the restaurant with probability 0.5 and carry their order without eating with probability 0.5. A meal requires an average of 20min. What is the average number of customers in the restaurant? Show your working.

Half eat in and half eat out, so let T1 reflect the arrival rate of those eating out, and T2 be those eating in

λ1= 5 \*0.5 = 2.5

λ2= 5 \*0.5 = 2.5

N = λ \* T = λ1\*T1 + λ2\*T2 = 2.5\*5+2.5\*(5+25) = 75

1. Customers arrive at a service counter at a rate of 0.5 per minute (assume Poisson arrivals) and service time is exponentially distributed with mean 90 seconds. a. (5pt)
2. What is the probability that the service counter is idle?

λ = 0.5 = 1/1.5min = 2/3

P0 = (1- λ/) = 1 – 0.75 = 0.25

1. (10pt) What is the average number of customers in the system

A: = λ/ = ¾

N = /（1-）= 3

1. What is the probability there are more than 3 customers in the system?

P(N>3) = 1 – P(N<=3) = 1 – P(N=0) – P(N=1) – P(N=2) -P(N=3)

= 1 – ¼ - (¾)^1\*¼ -(¾)^2\*¼ - (¾)^3\*¼ = 0.31640625

1. Using the Erlang B table (available on www.pitt.edu/~dtipper/2110/erlang-table.pdf), answer the following questions:

a (5pt) There are 60 servers and the target blocking probability is 1%. What is the largest load that can be supported?



46.95

b. Load is 30 Erlang and the target blocking probability is to be 10. How many servers are needed?

c There are 40 servers and the load is 25 Erlang. What is the expected blocking probability?

0.1%

5. Equation 3.4.2 in the Data Networks textbook (2-Queueing\_Data\_Nets.pdf in IVLE workbin) is for a M/G/1 system. Assume that arrival is a Poisson process with mean 0.1. For each of the following cases, calculate the average waiting time in the system for:

λ = 0.1 = E(X) = 1/ T = + λ\*/2\*(1-)

a. (10pt) service time is exponentially distributed with mean 5;

For exponentially distributed

Var(X) = E(X^2) − (E(X))2 = 1/λ^2.

E(X^2) = 2/λ^2 = 25

T = 10

b. (10pt) service time is always 5;

If Xi always 5 E(X^2) = 0

T = 5

c. (10pt) service times can be either 1 or 9 with equal probability.

1 and 9 with same probability 0.5

E(X^2) = 1^2\*0.5+9^2\*0.5 = 41

T = 9.1